HW-Binomial Distribution

1. The random variable $X$ has a binomial distribution with $n = 10$ and $p = 0.5$. Sketch the probability mass function of $X$.
   (a) What value of $X$ is most likely?
   (b) What value(s) of $X$ is(are) least likely?

2. The random variable $X$ has a binomial distribution with $n = 10$ and $p = 0.5$. Determine the following probabilities:
   (a) $P(X = 5)$
   (b) $P(X \leq 2)$
   (c) $P(X \geq 9)$
   (d) $P(3 \leq X < 5)$

3. Sketch the probability mass function of a binomial distribution with $n = 10$ and $p = 0.01$ and comment on the shape of the distribution.
   (a) What value of $X$ is most likely?
   (b) What value of $X$ is least likely?

4. An electronic product contains 40 integrated circuits. The probability that any integrated circuit is defective is 0.01, and the integrated circuits are independent. The product operates only if there are no defective integrated circuits. What is the probability that the product operates?

5. The phone lines to an airline reservation system are occupied 40% of the time. Assume that the events that the lines are occupied on successive calls are independent. Assume that 10 calls are placed to the airline.
   (a) What is the probability that for exactly three calls the lines are occupied?
   (b) What is the probability that for at least one call the lines are not occupied?
   (c) What is the expected number of calls in which the lines are all occupied?