Review for EXAM 1

Two events, denoted as $E_1$ and $E_2$, such that

$$E_1 \cap E_2 = \emptyset$$

are said to be mutually exclusive.

$$(A \cup B)' = A' \cap B' \quad \text{and} \quad (A \cap B)' = A' \cup B'$$

$$(A \cup B) \cap C = (A \cap C) \cup (B \cap C), \quad \text{and} \quad (A \cap B) \cup C = (A \cup C) \cap (B \cup C)$$

$$P(A \cap B) = P(A \mid B)P(B) = P(B \cap A) = P(B \mid A)P(A)$$

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)} \quad \text{for} \quad P(B) > 0$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

---

$$P(D \mid S) = P(S \mid D)P(D) / [P(S \mid D)P(D) + P(S \mid D')P(D')]$$

Probability is a number that is assigned to each member of a collection of events from a random experiment that satisfies the following properties:

If $S$ is the sample space and $E$ is any event in a random experiment,

1. $P(S) = 1$
2. $0 \leq P(E) \leq 1$
3. For two events $E_1$ and $E_2$ with $E_1 \cap E_2 = \emptyset$

$$P(E_1 \cup E_2) = P(E_1) + P(E_2)$$

$$P(\emptyset) = 0$$

$$P(E') = 1 - P(E)$$

if the event $E_1$ is contained in the event $E_2$, $P(E_1) \leq P(E_2)$
If $A$ and $B$ are mutually exclusive events,

$$P(A \cup B) = P(A) + P(B)$$

$$P(A \cup B \cup C) = P[(A \cup B) \cup C] = P(A \cup B) + P(C) - P[(A \cup B) \cap C]$$

$$= P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C)$$

$$- P(B \cap C) + P(A \cap B \cap C)$$

A collection of events, $E_1, E_2, \ldots, E_k$, is said to be mutually exclusive if for all pairs $E_i \cap E_j = \emptyset$

For a collection of mutually exclusive events,

$$P(E_1 \cup E_2 \cup \ldots \cup E_k) = P(E_1) + P(E_2) + \ldots + P(E_k)$$

For any events $A$ and $B$,

$$P(B) = P(B \cap A) + P(B \cap A^c) = P(B|A)P(A) + P(B|A^c)P(A^c)$$

Assume $E_1, E_2, \ldots, E_k$ are $k$ mutually exclusive and exhaustive sets. Then

$$P(B) = P(B \cap E_1) + P(B \cap E_2) + \cdots + P(B \cap E_k)$$

$$= P(B|E_1)P(E_1) + P(B|E_2)P(E_2) + \cdots + P(B|E_k)P(E_k)$$

Two events are independent if any one of the following equivalent statements is true:

1. $P(A|B) = P(A)$
2. $P(B|A) = P(B)$
3. $P(A \cap B) = P(A)P(B)$
The events \( E_1, E_2, \ldots, E_n \) are independent if and only if for any subset of these events \( E_{i_1}, E_{i_2}, \ldots, E_{i_k} \),

\[
P(E_{i_1} \cap E_{i_2} \cap \cdots \cap E_{i_k}) = P(E_{i_1}) \times P(E_{i_2}) \times \cdots \times P(E_{i_k})
\]

Assume that the probability that a wafer contains a large particle of contamination is 0.01 and that the wafers are independent; that is, the probability that a wafer contains a large particle is not dependent on the characteristics of any of the other wafers. If 15 wafers are analyzed, what is the probability that no large particles are found?

Let \( E_i \) denote the event that the \( i \)th wafer contains no large particles,

\[ P(E_i) = 0.99 \]

\[ P(E_1 \cap E_2 \cap \cdots \cap E_{15}) = P(E_1) \times P(E_2) \times \cdots \times P(E_{15}) = 0.99^{15} = 0.86 \]

A visual inspection of a location on wafers from a semiconductor manufacturing process resulted in the following table:

<table>
<thead>
<tr>
<th>Number of Contamination Particles</th>
<th>Proportion of Wafers</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.40</td>
</tr>
<tr>
<td>1</td>
<td>0.20</td>
</tr>
<tr>
<td>2</td>
<td>0.15</td>
</tr>
<tr>
<td>3</td>
<td>0.10</td>
</tr>
<tr>
<td>4</td>
<td>0.05</td>
</tr>
<tr>
<td>5 or more</td>
<td>0.10</td>
</tr>
</tbody>
</table>

What is the probability that a wafer contains three or more particles in the inspected location? Let \( E \) denote the event that a wafer contains three or more particles in the inspected location. Then, \( E \) consists of the three outcomes \( \{3, 4, 5 \text{ or more}\} \). Therefore,

\[
P(E) = 0.10 + 0.05 + 0.10 = 0.25
\]
A day’s production of 850 manufactured parts contains 50 parts that do not meet customer requirements. Two parts are selected randomly without replacement from the batch. What is the probability that the second part is defective given that the first part is defective?

Let $A$ denote the event that the first part selected is defective, and let $B$ denote the event that the second part selected is defective. The probability needed can be expressed as $P(B|A)$. If the first part is defective, prior to selecting the second part, the batch contains 849 parts, of which 49 are defective, therefore

$$P(B|A) = \frac{49}{849}$$

If three parts are selected at random, what is the probability that the first two are defective and the third is not defective? This event can be described in shorthand notation as simply $P(ddn)$. We have

$$P(ddn) = \frac{50}{850} \cdot \frac{49}{849} \cdot \frac{800}{848} = 0.0032$$

If $E_1, E_2, \ldots, E_k$ are $k$ mutually exclusive and exhaustive events and $B$ is any event,

$$P(E_1|B) = \frac{P(B|E_1)P(E_1)}{P(B|E_1)P(E_1) + P(B|E_2)P(E_2) + \cdots + P(B|E_k)P(E_k)}$$

for $P(B) > 0$

$$P(D|S) = \frac{P(S|D)P(D)}{P(S|D)P(D) + P(S|D')P(D')}$$